Simply Irrational

Ujwal N K
Departement of Electrical and Electronics Engineering, PES University, Bangalore.
ujwal.nk@gmail.com

Abstract

Irrational numbers, though uncommon, are indispensable for certain scientific calculations. Rational numbers can be represented in multiple forms, but representing rational numbers in irrational form was never tried before.

In this paper, we look into the different aspects of this representation.

1 Introduction

In the past, there have been multiple methods of representing rational numbers like [1], Various methods to fit rational and irrational numbers on real number lines [2].

Due to the nature of certain decimals and the complex nature of irrational numbers, there has not been an attempt to express rational numbers and decimals using the form expressed in this paper. In this paper, we explore a new representation for such decimal numbers to its nearest possible closed irrational form.

The need for such forms appears in various fields like spectral analysis and spectral graph theory.

The energy of a simple connected graph G is equal to the sum of the absolute value of the eigenvalues of its adjacency matrix given by,

$$E(G) = \Sigma|\lambda_i|$$

.

Sometimes it is difficult to obtain the general form of energy corresponding to the molecular structure of hydrocarbons due to the decimal form of eigenvalues obtained from its adjacency matrix. In such cases, the irrational form of the decimal gives the closed form of energy, which is easy for computation.

2 Methodology

To get the nearest irrational form of a given decimal, the algorithm proposed works on brute force method.

3 Proposition

Every decimal number $'a.bc'$ where $\forall a \in Z \;\; and \;\; b, c \in \{0, 1, 2...9\}$ can be expressed uniquely in the form

$$x \pm \sqrt{y}, \qquad \forall x \in Z, y \in Z^+ - \{n^2 | n \in W\}. \quad (1)$$

Through systematic exploration and exhaustive testing, it has been found that for any decimal number 'a.bc', there exist integers x and whole numbers y, such that the above expression is satisfied. This representation offers a concise and structured way to express $a.bc$.

3.1 Algorithm

- Determine the least whole number y such that $\sqrt{y} = a' + 0.bc$ or $\sqrt{y} = a' + (1 - 0.bc)$
- This determines the value of $x = a + a'$
- Since this can lead to multiple possible combinations of x and y for a given decimal, to get the unique expression, choose the smallest possible value of y

However, as the precision is increased to 3 or higher, it has been discovered that not all possible combinations of decimal numbers are covered by this form. This observation suggests that there might be inherent limitations to the specific form of (1) when expressing decimal numbers with higher precision.

Remark 1: This representation does not claim to be the only possible form for expressing decimal numbers. There may be alternative representations or transformations. Remark 2: This algorithm can be extended as

$$x \pm \sqrt[c]{y}, \qquad \forall x \in Z, y \in Z^+ - \{n^c | n \in W\}. \quad (2)$$

4 Increasing the precision

In scientific calculations, the need for higher precision arises. This gives rise to the question of how to increase the precision beyond two decimal places. One possible solution is to utilize scientific notation

$$\alpha = 9.9086897273026$$

Let us consider the above example with a desired precision of four digits. This is not possible with the current expression, but in this case, we can express the number using scientific notation as

$$\alpha = 908.68 \times 10^{-2}$$

Thus, we can employ this expression for simplification.

$$\alpha = (912 - \sqrt{11}) \times 10^{-2}$$

By employing scientific notation, we maintain the same general form with the constraint of a precision limit of two decimal places.

However, adhering strictly to the requirement of maintaining precisely two decimal places can lead to a deviation from the conventional notation that follows standard powers of 10, such as milli, micro, and nano.

5 Illustration

Consider a simple Graph G with n vertices whose adjacency matrix has the following representation. For $n = 10$

$$\begin{pmatrix}
01111111111000000000 \\
10111111111000000000 \\
11011111111000000000 \\
11101111111000000000 \\
11110111111000000000 \\
11111011111000000000 \\
11111101111000000000 \\
11111110111000000000 \\
11111111011000000000 \\
11111111101111111111 \\
11111111110111111111 \\
00000000011011111111 \\
00000000011101111111 \\
00000000011110111111 \\
00000000011111011111 \\
00000000011111101111 \\
00000000011111110111 \\
00000000011111111011 \\
00000000011111111101 \\
00000000011111111110
\end{pmatrix}$$

The spectrum corresponding to the above matrix is, $\begin{pmatrix} -2.1231 & -1 & 3 & 6.1231 \\ 1 & 7 & 1 & 1 \end{pmatrix}$

The decimals simplifies to: -(6-$\sqrt{15}$), $10-\sqrt{15}$

As we observe, there are eigen values that are decimal, to obtain a pattern for higher values of n with such a decimal representation, generalizing the closed form of energy would be a hassle. Thus, we can use the above algorithm to simplify the decimals which gives the closed form of energy easily.

For a few other values, the table is as follows.

n	EigenValues	Irrational form of decimal Eigenvalues	
10	$\begin{pmatrix} -2.1231 & -1 & 3 & 6.1231 \\ 1 & 7 & 1 & 1 \end{pmatrix}$	-(6-$\sqrt{15}$)	10 - $\sqrt{15}$
12	$\begin{pmatrix} -2.2170 & -1 & 4 & 7.2170 \\ 1 & 9 & 1 & 1 \end{pmatrix}$	-(9 - $\sqrt{46}$)	14 -$\sqrt{46}$
14	$\begin{pmatrix} -2.2915 & -1 & 5 & 8.2915 \\ 1 & 11 & 1 & 1 \end{pmatrix}$	-($\sqrt{28} - 3$)	$\sqrt{28} + 3$

Thus, with respect to the irrational form of the eigen values, we can generalize the characteristc polynomial for the above graph as follows:

$(\lambda^2 - (k+1)\lambda - (n+k))(\lambda + 1)^(n-3)(\lambda - k)$
$n = 2(k+1) + 2$
$k = 3, 4, ...$

6 Performance Analysis

As the decimals are converted to their respective irrational forms, these forms can be stored in an integer data type, this enables faster computation when computing the eigenvalues in the above form, as they get subtracted in integer format which is faster than a floating point operation.

7 Conjecture

- When does the need to decrease the power of y, taking into consideration the cube and fourth roots and n^{th} root.
- How to determine which decimal numbers have the possibility of irrational forms with higher precision.
- Mathematical proof for the uniqueness of the above-mentioned form (1).
- The possibility of utilizing these quantities for a graphical representation $(x, y, precision)$

- Algorithms for mathematical operations on numbers in the above form (1).

8 Conclusions

Every decimal number can be represented in a unique way using the expression (1).

References

[1] Yehuda Rav. "On the representation of rational numbers as a sum of a fixed number of unit fractions". In: (). DOI: 10.1515/crll.1966.222.207. URL: https://doi.org/10.1515/crll.1966.222.207.

[2] Natasa Sirotic and Andrina Zazkis. "Irrational Numbers: The Gap between Formal and Intuitive Knowledge". In: *Educational Studies in Mathematics* 65.1 (May 2007), pp. 49–76. ISSN: 1573-0816. DOI: 10.1007/s10649-006-9041-5. URL: https://doi.org/10.1007/s10649-006-9041-5.